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Are Constants Constant?

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The prospect of a time-dependent Higgs vacuum expectation value is examined within the standard model of electroweak interactions. It is shown that the classical equation of motion for the Higgs field admits a solution that is a doubly-periodic function of time. The corresponding Dirac equation for the electron field is equivalent to a second order differential equation with doubly-periodic coefficients. In the limit of very large primitive period of the Higgs background this equation can be solved in WKB approximation, showing plane-wave solutions with a time-dependent distortion factor which can be made arbitrarily small.

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1 Introduction

The possibility that the fundamental constants of nature might vary with time has been an object of speculations for many years [1]. In a modern language, however, one should say that the prospect of a time variation in the vacuum expectation value (hereafter vev) of the Higgs field seems more plausible than the time variation of the Fermi coupling constant G_F or of the electron mass m_e [2].

Changing the vev of the Higgs field has many physical effects, four of which have astrophysical consequences: G_F changes, the electron mass m_e changes and the nuclear masses and binding energies change. All of these effects alter Big Bang nucleosynthesis. On the other end the change in m_e is the only effect relevant for the cosmic microwave background spectrum [3].

How much do we know about possible variations of m_e ? We should remember that one of the recurring themes in the physics behind the fundamental constants is that their values are rarely determined by a direct measurement. The example of the electron mass illustrates how the information that leads to the values of the constants can be indirect and how different paths provide redundant constraints on their values. To obtain the best values, all of this information are taken into account simultaneously; in the approach of the 1998 adjustment [4], the information is divided into three categories: input data, observational equations, and adjusted constants. The observational equations are theoretical expressions that give values of the quantities in the input data category as functions of the adjusted constants. The adjusted constants are a suitably chosen set of fundamental constants that are determined by the adjustment. The adjustment's role is to find the values that best reproduce the input data by means of the theoretical expressions.

There are recent studies that show, for instance, how a change in m_e alters the CMB fluctuation spectrum. There it is assumed that the variation in m_e is sufficiently small during the process of recombination so that, one needs only consider the difference between m_e at recombination and m_e today, see [5] and also [6]. Furthermore, it has been pointed out [3] that MAP and PLANK experiments might be sensitive to variations as small as $|\Delta m_e/m_e| \sim 10^{-2} - 10^{-3}$.

Although all of these considerations look very promising, we are still missing an important ingredient in the discussion: can a time variation in the electron mass (any mass) be made formally consistent with the Standard Model of strong and electroweak interactions? In other words, do we have to assume an ad hoc time dependence in these parameters or do we have some explicit time variation which is consistent with the mathematical structure of the Standard Model? Furthermore, a time-dependent mass is a ill-defined concept since there is no stationary state in a time-dependent external field.

In this paper we show that the classical equation of motion for the Higgs field in the Minimal Standard Model admits a time-dependent solution which can be given in terms of a Jacobian elliptic function, therefore a doubly-periodic function of time. This solution is explicitly constructed in Sect. 2. In Sect. 3 we consider the coupling of fermionic fields with the Higgs sector and study the effect of their propagation in a time-dependent Higgs background. The three-momentum \mathbf{p} is conserved since the background depends only on time and we can factorize the usual term, $\exp(i \mathbf{p} \cdot \mathbf{x})$. In the limit of very large primitive period of the Higgs background the Dirac equation

for the electron field can be solved in WKBJ approximation. The solution can be cast into the form of a plane-wave with a time-dependent distortion factor which becomes arbitrarily small exactly in the limit of infinite primitive period of the Higgs background.

2 A time dependent solution for the Higgs vev

The relevant part of the Standard Model Lagrangian that we are interested in is [7]

$$\mathcal{L}_H = -\partial_\mu K^\dagger \partial_\mu K - \mu^2 K^\dagger K - \frac{1}{2} \lambda (K^\dagger K)^2, \quad (1)$$

where K is a complex iso-doublet,

$$K = \frac{1}{\sqrt{2}} (\psi + i\phi^a \tau^a) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (2)$$

and where we have neglected gauge couplings. We introduce two new quantities as follows:

$$M_H^2 = 4 \frac{\lambda}{g^2} M^2, \quad \mu^2 = \beta - \frac{1}{2} M_H^2, \quad (3)$$

where g is the $SU(2)$ coupling constant and, for a constant vev, M_H is the bare Higgs boson mass. As usual we perform a shift in the ψ -field

$$\psi = 2 \frac{M}{g} \Phi + H. \quad (4)$$

The parameter β will be adjusted in perturbation theory so that, order-by-order, $\langle 0|H|0 \rangle = 0$. The well known solution which is at the basis of the so-called spontaneous symmetry breaking mechanism is $\Phi^2 = 1$. For a constant, non-zero, value of $\langle 0|\psi|0 \rangle = M$ is the bare W -boson mass and M_H the bare Higgs boson mass. In the present case we allow Φ to be function of time and arrive at the following equation, where according to the usual procedure $\beta = 0$ at tree level:

$$\frac{d^2}{dt^2} \Phi - \frac{1}{2} M_H^2 \Phi (1 - \Phi^2) = 0. \quad (5)$$

A solution to this equation is given in terms of Jacobian elliptic functions [8],

$$\begin{aligned} \Phi(t) &= N_k \operatorname{cn}(H_k t, k), \\ N_k^2 &= \frac{2k^2}{L_k^2}, \quad L_k^2 = 2k^2 - 1, \quad H_k = \frac{M_H}{\sqrt{2} L_k}, \end{aligned} \quad (6)$$

and k is any positive real number. Note that we adopt the definition

$$\begin{aligned} u &= \int_0^\phi dt (1 - k^2 \sin^2 t)^{-1/2}, \quad \phi = \operatorname{am}(u), \\ \operatorname{cn}(u) &= \operatorname{cn}(u, k) = \cos(\operatorname{am}(u)), \quad \operatorname{sn}(u) = \operatorname{sn}(u, k) = \sin(\operatorname{am}(u)), \\ \operatorname{dn}(u) &= \operatorname{dn}(u, k) = [1 - k^2 \sin^2(\operatorname{am}(u))]^{1/2}. \end{aligned} \quad (7)$$

Such solutions are well known in the literature for both (1+1)-dimensional and (3+1)-dimensional variants of ϕ^4 theory [9]. The functions $\text{cn}(z, k)$, $\text{dn}(z, k)$ have the following properties:

$$\begin{aligned} \text{periods : } & 4\mathbf{K}, 2\mathbf{K} + 2i\mathbf{K}' & 2\mathbf{K}, 4i\mathbf{K}' \\ \text{zeros : } & (2m+1)\mathbf{K} + 2ni\mathbf{K}' & (2m+1)\mathbf{K} + (2n+1)i\mathbf{K}' \\ \text{poles : } & \beta_{mn} = 2m\mathbf{K} + (2n+1)i\mathbf{K}' \end{aligned} \quad (8)$$

where n, m are integers and $\mathbf{K}(k)$ is the complete elliptic integral of the first kind

$$\mathbf{K}(k) = F\left(\frac{\pi}{2}, k\right), \quad \mathbf{K}'(k) = F\left(\frac{\pi}{2}, k'\right), \quad (9)$$

with $k' = (1 - k^2)^{1/2}$. Clearly we are interested in solutions with large values of k since in that limit

$$\text{cn}(H_k, t, k) = \text{dn}\left(\tau, \frac{1}{k}\right) \sim 1 - \frac{1}{2k^2} \sin^2 \tau + \mathcal{O}\left(\frac{1}{k^4}\right), \quad \tau = k H_k t, \quad (10)$$

and the vev of the Higgs field is approximately constant with time periodic fluctuations suppressed by a factor k^{-2} . The Lagrangian becomes

$$\mathcal{L}_H = -\frac{1}{2} \partial_\mu H \partial_\mu H + 2 \frac{M^2}{g^2} \left[\dot{\Phi}^2 - \left(\beta - \frac{1}{2} M_H^2 \right) \Phi^2 - \frac{1}{4} M_H^2 \Phi^4 \right] + \mathcal{O}(H^2). \quad (11)$$

This result, and the vev of the H -field in higher orders of perturbation theory require some additional comment. There is a total derivative to be considered,

$$2 \frac{M}{g} k^2 H_k^2 \frac{d}{d\tau} \left(H \frac{d\Phi}{d\tau} \right). \quad (12)$$

Therefore, to quantize the H -field, we need to consider a time interval $\{-\tau_L, +\tau_R\}$

$$\tau_L = (2m+3)\mathbf{K}, \quad \tau_R = (2m+1)\mathbf{K}, \quad (13)$$

where m is an integer, $4\mathbf{K}$ is the primitive period of $\dot{\Phi}$ and $\dot{\Phi}(-\tau_L) = \dot{\Phi}(\tau_R) = 0$. For the quantum fluctuations H we impose periodic boundary conditions. At the end the limit $m \rightarrow \infty$ will be taken.

In this paper, however, we are not so much interested in the Higgs sector, i.e. Higgs mass and Higgs self-couplings, or in the problem of time dependence of the cosmological constant but rather we concentrate on the effect of a time-depended Higgs vev on the *masses* of elementary particles. For this reason we will analyze in the following section the Higgs-fermion Yukawa couplings.

3 Higgs-fermion interaction

The relevant piece of the Lagrangian for arbitrary u, d fermion fields will be [7]

$$\mathcal{L}_{H-f} = -\bar{\psi}_L \not{\partial} \psi_L - \bar{\psi}_R \not{\partial} \psi_R + \frac{1}{\sqrt{2}} g \frac{m_d}{M} \bar{\psi}_L K^c d_R - \frac{1}{\sqrt{2}} g \frac{m_u}{M} \bar{\psi}_L K u_R + \text{h.c.}, \quad (14)$$

where ψ_L is a left-handed doublet and $u(d)_R$ are right-handed singlets of $SU(2)$ with K^c being the charge-conjugate of K . For Φ constant m_u, m_d are just the bare up, down masses. For the (ν_e, e) doublet we find

$$\mathcal{L}_m = -\bar{\nu}_e \not{\partial} \nu_e - \bar{e} \not{\partial} e - \frac{1}{2} g \frac{m_e}{M} \psi^* \bar{e} e. \quad (15)$$

Here m_e is an arbitrary free parameter with dimension of mass which cannot be identified with the bare electron mass since the fermion fields are moving in a time-dependent background. The neutrino remains decoupled from the Higgs field while the Dirac equation for the electron becomes

$$(\not{\partial} + m_e \Phi) e = 0. \quad (16)$$

If we split the field e into upper/lower two-dimensional components, e_+/e_- , we obtain the following equations:

$$i \lambda \sigma^a \partial_a e_\lambda + i \lambda \partial_t e_{-\lambda} + m_e \Phi e_{-\lambda} = 0, \quad (17)$$

where σ^a , $a = 1, 2, 3$ are Pauli matrices and $\lambda = \pm 1$. To find a solution we introduce

$$e_\lambda = e^{i p^a x_a} \chi_\lambda, \quad (18)$$

and derive the corresponding equation for χ_λ ,

$$\sigma^a p_a \chi_\lambda - i \partial_t \chi_{-\lambda} - \lambda m_e \Phi \chi_{-\lambda} = 0. \quad (19)$$

The above equation is solved by introducing a new set of variables, scalar and vector modes:

$$\chi_\lambda = \frac{1}{\sqrt{2}} (S_\lambda + i V_\lambda^a \sigma_a) \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (20)$$

Next we introduce a set of matrices $\sigma^a(p)$, such that

$$p_\pm = \frac{1}{\sqrt{2}} (p_x \mp i p_y), \quad p = |\mathbf{p}|, \\ U_{ii} = N, \quad i = 1, 2, \quad U_{12(21)} = \pm \sqrt{2} N \frac{p_\pm}{p + p_z}, \quad N^{-2} = 2 \frac{p}{p + p_z}. \quad (21)$$

With this definition it is easy to prove [10] that the matrices

$$\sigma^a(p) = U^\dagger \sigma^a U, \quad (22)$$

satisfy the following properties:

$$\sigma^3(p) = \sigma \cdot \mathbf{p}, \quad \sigma^{1,2}(p) = \sigma \cdot \hat{e}_{1,2}, \quad (23)$$

with $\mathbf{e}_3 = \mathbf{p}/p$ and

$$\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}, \quad \mathbf{e}_i \times \mathbf{e}_j = \varepsilon_{ijk} \mathbf{e}_k, \quad (24)$$

Using these matrices we derive a useful decomposition for the vector modes,

$$\begin{aligned}\mathbf{V} &= V_L \mathbf{e}_3 + \sum_{i=1,2} V_\perp^i \mathbf{e}_i, \\ V_\lambda^a \sigma_a &= V_L(\lambda) \sigma^3(p) + \sum_{i=1,2} V_\perp^i(\lambda) \sigma_i(p).\end{aligned}\tag{25}$$

By equating the coefficients of $\mathbf{1}$ and $\sigma(p)$ we obtain two separate systems of equations relative to the SL (scalar-longitudinal) and T (transverse) modes:

$$\begin{aligned}\frac{d}{dt} S(\lambda) + i \lambda m_e \Phi S(\lambda) - p V_L(-\lambda) &= 0, \\ \frac{d}{dt} V_L(\lambda) + i \lambda m_e \Phi V_L(\lambda) + p S(-\lambda) &= 0, \\ \frac{d}{dt} V_\perp^{1,2}(\lambda) + i \lambda m_e \Phi V_\perp^{1,2}(\lambda) \pm p V_\perp^{2,1}(-\lambda) &= 0.\end{aligned}\tag{26}$$

Solutions of the Dirac equation are classified as follows:

$$\chi_\lambda^{SL} = \frac{1}{\sqrt{2}} [S(\lambda) + i V_L(\lambda) \mathbf{e}_3 \cdot \sigma] \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_\lambda^T = \frac{i}{\sqrt{2}} \sum_{i=1,2} V_\perp^i(\lambda) \mathbf{e}_i \cdot \sigma \begin{pmatrix} 1 \\ 0 \end{pmatrix}.\tag{27}$$

A solution to Eq.(26) is obtained by introducing a function a_λ such that

$$\begin{aligned}S(\lambda) &= a_\lambda f_\lambda, \quad V_L(-\lambda) = \frac{a_\lambda}{p} \frac{df_\lambda}{dt}, \\ \frac{da_\lambda}{dt} &= -i \lambda m_e \Phi a_\lambda,\end{aligned}\tag{28}$$

and similarly for $V_\perp^{1,2}(\lambda)$, i.e.

$$V_\perp^2(\lambda) = a_\lambda f_\lambda, \quad V_\perp^1(-\lambda) = \frac{a_\lambda}{p} \frac{df_\lambda}{dt}.\tag{29}$$

Here the Higgs background is rewritten as

$$\begin{aligned}\Phi &= N_k \operatorname{cn}(H_k t, k) = N_k \operatorname{dn}\left(\tau, \frac{1}{k}\right), \\ \tau &= \frac{k M_H t}{\sqrt{2} L_k} \sim \frac{1}{2} M_H t, \quad \text{for } k \rightarrow \infty,\end{aligned}\tag{30}$$

using a relation that is based on the Jacobi's real transformation

$$\begin{aligned}\operatorname{cn}(H_k t, k) &= \operatorname{dn}\left(\tau, \frac{1}{k}\right), \\ \operatorname{dn}(H_k t, k) &= \operatorname{cn}\left(\tau, \frac{1}{k}\right), \\ \operatorname{sn}(H_k t, k) &= k^{-1} \operatorname{sn}\left(\tau, \frac{1}{k}\right).\end{aligned}\tag{31}$$

Moreover, for $k \rightarrow \infty$, the following approximations hold:

$$\begin{aligned}\operatorname{cn}\left(\tau, \frac{1}{k}\right) &\sim \cos \tau + \frac{1}{4k^2} \left(\tau - \frac{1}{2} \sin 2\tau\right) \sin \tau, \\ \operatorname{dn}\left(\tau, \frac{1}{k}\right) &\sim 1 - \frac{1}{2k^2} \sin^2 \tau, \\ \operatorname{sn}\left(\tau, \frac{1}{k}\right) &\sim \sin \tau - \frac{1}{4k^2} \left(\tau - \frac{1}{2} \sin 2\tau\right) \cos \tau.\end{aligned}\tag{32}$$

With $r = m_e/M_H$ we obtain a solution for a_λ ,

$$\begin{aligned}a_\lambda(\tau) &= \exp\left\{-2i\lambda r \arcsin\left[\operatorname{sn}\left(\tau, \frac{1}{k}\right)\right]\right\}, \\ &\sim \exp\left\{-2i\lambda r \tau + \mathcal{O}\left(\frac{1}{k}\right)\right\} = \exp\left\{-i\lambda m_e t + \mathcal{O}\left(\frac{1}{k}\right)\right\}.\end{aligned}\tag{33}$$

The function f , therefore, satisfies the following differential equation:

$$\frac{d^2 f_\lambda}{d\tau^2} - 4i\lambda r \operatorname{dn}\left(\tau, \frac{1}{k}\right) \frac{df_\lambda}{d\tau} + q^2 f_\lambda = 0,\tag{34}$$

where we have introduced a new parameter

$$q^2 = 2 \frac{L_k^2}{k^2} \frac{p^2}{M_H^2}.\tag{35}$$

The function dn is doubly-periodic with periods $2\mathbf{K}$ and $4i\mathbf{K}'$. Due to the periodicity we can discuss all properties of the elliptic function in the so-called fundamental period parallelogram which for dn is $\tau = 2\xi\mathbf{K} + 4i\eta\mathbf{K}'$ with $0 \leq \xi, \eta < 1$. An irreducible set of poles is given by

$$\beta = \beta_{00} = i\mathbf{K}', \quad \beta' = \beta_{01} = 3i\mathbf{K}',\tag{36}$$

with residues $-i$ and $+i$ respectively. Therefore, the singular points of the second order differential equation for f_λ are $\tau = \beta, \beta'$ and their congruent points. The corresponding exponents are $0, 1 \pm 4\lambda r$ for $\beta(\beta')$. We know that Eq.(34) possesses a fundamental set of solutions but, unfortunately, the exponents are not unequal integers and, therefore, we cannot apply the Hermite, Picard, Mittag-Leffler, Floquet theorem [8] stating that the solutions are doubly-periodic functions of second kind and, in general, expressible as products of ratios of weierstrassian σ -functions, see also [11].

We have not been able to find an exact, explicit, solution to Eq.(34) but an approximated one can be given by using the WKBJ method, based on the observation that, for $k \rightarrow \infty$,

$$\begin{aligned}\dot{\Phi} &= -N_k H_k \operatorname{dn}(H_k t, t) \operatorname{sn}(H_k t, k) \sim -\frac{1}{2k^2} M_H \cos \tau \sin \tau, \\ \Phi &= N_k \operatorname{cn}(H_k t, k) \sim 1 - \frac{1}{2k^2} \sin^2 \tau.\end{aligned}\tag{37}$$

Therefore, in this limit, we obtain

$$\begin{aligned} f_\lambda^\pm(\tau) &= \exp\left\{i \int_0^\tau du \Theta_\lambda^\pm(u)\right\}, \\ \Theta_\lambda^\pm(u) &= 2\lambda \operatorname{r} \operatorname{dn}\left(u, \frac{1}{k}\right) \pm \left[q^2 + 4\operatorname{r}^2 \operatorname{dn}^2\left(u, \frac{1}{k}\right)\right]^{1/2}. \end{aligned} \quad (38)$$

The integrals appearing in Eq.(38) give

$$\begin{aligned} \int_0^\tau du \operatorname{dn}\left(u, \frac{1}{k}\right) &= \arcsin\left(\operatorname{sn}\left(\tau, \frac{1}{k}\right)\right) \sim \tau - \frac{1}{4k^2} \left(\tau - \frac{1}{2} \sin 2\tau\right) + \mathcal{O}(k^{-4}), \\ \int_0^\tau du \left[q^2 + 4\operatorname{r}^2 \operatorname{dn}^2\right]^{1/2} &= \int_0^\tau du \left[4\mathcal{R} - h^2 \sin^2 u + \mathcal{O}(k^{-4})\right]^{1/2}, \\ &= 2\sqrt{\mathcal{R}} E\left(\tau, \frac{h}{2\sqrt{\mathcal{R}}}\right) + \mathcal{O}(k^{-4}) \\ &= 2\sqrt{\mathcal{R}} \tau - \frac{1}{8} \frac{h^2}{\sqrt{\mathcal{R}}} \left(\tau - \frac{1}{2} \sin(2\tau)\right) + \mathcal{O}(h^4), \end{aligned} \quad (39)$$

where we have introduced the following parameters,

$$\mathcal{R} = \frac{1}{4}q^2 + \operatorname{r}^2 = \frac{p^2 + m_e^2}{M_H^2} + \mathcal{O}(k^{-2}), \quad h = 2\frac{r}{k}, \quad (40)$$

and where E is the elliptic integral of second kind. In the limit $k \rightarrow \infty$ we also have

$$\mathcal{E}^2 = p^2 + m_e^2, \quad \mathcal{R} = \frac{\mathcal{E}^2}{M_H^2} - \frac{p^2}{2M_H^2 k^2}, \quad 2\sqrt{\mathcal{R}}\tau \sim \mathcal{E}t. \quad (41)$$

To summarize, in WKBJ approximation, we find the following result for Θ of Eq.(38),

$$\Theta_\lambda^\pm(u) = \lambda \operatorname{r} \left[2 - \frac{1}{k^2} \sin^2 u\right] \pm 2\sqrt{\mathcal{R}} \left(1 - \frac{h^2}{2\mathcal{R}} \sin^2 u\right) + \mathcal{O}(k^{-4}). \quad (42)$$

Note that in the product $a_\lambda f_\lambda$ (see Eq.(33)) $\arcsin(\operatorname{sn}(\tau, 1/k))$ drops out and, by interacting with the time-dependent Higgs background, the positive(negative) energy plane wave-solutions that would correspond to a free electron of mass m_e ,

$$\exp\{\pm i \mathcal{E} t\}, \quad (43)$$

receive a distortion factor which can be made arbitrarily small for large values of k . From Eq.(28) we derive

$$\begin{aligned} S^\pm(\lambda) &= a_\lambda f_\lambda^\pm = \exp\left\{i \int_0^\tau du \Theta_\lambda^\pm(u)\right\}, \\ V_L^\pm(-\lambda) &= i S^\pm(\lambda) \left\{\lambda \frac{m_e}{p} \left(1 - \frac{\sin^2 \tau}{2k^2}\right) \pm \frac{\mathcal{E}}{p} \left[1 - \frac{p^2}{4\mathcal{E}^2 k^2} - \frac{m_e^2}{2\mathcal{E}^2 k^2} \sin^2 \tau\right]\right\}. \end{aligned} \quad (44)$$

A similar results holds for the transverse components $V_{\perp}^{1,2}(\lambda)$. A more accurate version of the approximated solution will now be derived. Starting from Eq.(34) we write

$$f = \exp(\theta) F, \quad \theta = 2 i \lambda r \operatorname{am}(\tau), \quad (45)$$

with $\operatorname{am}(\tau)$ defined in Eq.(7) and where F is a solution of

$$\frac{d^2 F_{\lambda}}{d\tau^2} + \left[q^2 + 4 r^2 \operatorname{dn}^2 \left(\tau, \frac{1}{k} \right) - 2 i \lambda \frac{r}{k^2} \operatorname{sn} \left(\tau, \frac{1}{k} \right) \operatorname{cn} \left(\tau, \frac{1}{k} \right) \right] F_{\lambda} = 0. \quad (46)$$

The standard WKBJ solution of the above equation, based on the fact that Φ is a slowly varying function of τ , follows by introducing

$$Q_{\lambda}^2(u) = q^2 + 4 r^2 - 4 \frac{r^2}{k^2} \sin^2 u - 2 i \lambda \frac{r}{k^2} \sin u \cos u + \mathcal{O}(k^{-4}). \quad (47)$$

We easily derive a solution for F ,

$$F_{\lambda}^{\pm}(\tau) \sim \frac{r}{Q_{\lambda}(\tau)} \exp \left\{ \pm i \int_0^{\tau} du Q_{\lambda}(u) \right\}, \quad (48)$$

where the integral in the exponent gives

$$\int_0^{\tau} du Q_{\lambda}(u) = 2 \sqrt{\mathcal{R}} \tau - \frac{1}{k^2} \frac{r^2}{\sqrt{\mathcal{R}}} \left(\tau - \frac{1}{2} \sin(2\tau) \right) + \frac{i}{8 k^2} \frac{\lambda r}{\sqrt{\mathcal{R}}} (\cos(2\tau) - 1) + \mathcal{O}(k^{-4}). \quad (49)$$

Note that for $p^2 = 0$ we have an exact solution of Eq.(26). Let f_{λ} be any of the functions $S(\lambda)$, $V_L(\lambda)$, or $V_{\perp}^i(\lambda)$. The corresponding equation becomes

$$\frac{df_{\lambda}}{d\tau} + 2 i \lambda r \operatorname{dn} \left(\tau, \frac{1}{k} \right) f_{\lambda} = 0, \quad (50)$$

with a solution

$$f_{\lambda} = \exp \left\{ -2 i \lambda r \operatorname{arcsin} \left[\operatorname{sn} \left(\tau, \frac{1}{k} \right) \right] \right\} = \exp \left\{ -2 i \lambda r \operatorname{am}(\tau) \right\}, \quad (51)$$

with $\operatorname{am}(\tau)$ defined in Eq.(7). Expanding for large values of k^2 gives

$$\operatorname{am}(\tau) = \tau - \frac{1}{4 k^2} (\tau - \sin \tau \cos \tau) + \mathcal{O}(k^{-4}). \quad (52)$$

Since $2 r \tau \sim m_e t$ we again recover the correct limit of the free Dirac equation. We arrive at the same conclusion by writing Eq.(16) as

$$\frac{d^2 e}{dt^2} + i m_e \dot{\Phi} \gamma^4 e + m_e^2 \Phi^2 e = 0, \quad (53)$$

and expanding the spinor $e(t)$

$$e = \sum_{i=1,4} e^i(t) u_i, \quad (54)$$

where the u are eigenfunctions of the spin generator. It follows

$$\frac{d^2 e^i}{d\tau^2} + 4r^2 \operatorname{dn}\left(\tau, \frac{1}{k}\right) e^i \pm 2i \frac{r}{k^2} \operatorname{cn}\left(\tau, \frac{1}{k}\right) \operatorname{sn}\left(\tau, \frac{1}{k}\right) e = 0, \quad (55)$$

where the \pm refers to $i = 1, 2$ and $i = 3, 4$ respectively. Using Eq.(47) we easily derive, in WKB approximation,

$$e^{1,2} = \frac{r}{Q_-(\tau)} \exp\left\{-i \int_0^\tau du Q_-(u)\right\}, \quad e^{3,4} = \frac{r}{Q_+(\tau)} \exp\left\{+i \int_0^\tau du Q_+(u)\right\}. \quad (56)$$

In the above result it is understood that $p = 0$ and $\sqrt{\mathcal{R}} = r$.

4 Conclusions

In this paper we have tried to answer an important question related to the possibility that the fundamental constants of nature might vary with time: can a time variation in the electron mass (any mass) be made formally consistent with the structure of the Standard Model of strong and electroweak interactions? For an exhaustive study of how a change in m_e alters physical effects we refer to [3].

We have shown that the classical equation of motion for the Higgs field in the Minimal Standard Model admits a time-dependent solution which can be given in terms of the Jacobian elliptic function cn which is a doubly-periodic function of time. Therefore, we have a consistent picture of a time-dependent vev of the Higgs field. The background is not an external field but rather it is derived directly from the equations of motion.

By choosing the primitive period of the Higgs vev large enough we have a picture of the vacuum that is not in any evident contradiction with plain experimental evidence. By this we mean that the present value of the proton-to-electron mass ratio is $\mu = 1836.1526645(57)$ and any *significant* variation of this parameter over a small time interval is excluded ($\Delta\mu/\mu < 2 \times 10^{-4}$ [12]).

Next we have considered the coupling of fermions with the Higgs sector of the Minimal Standard Model and have studied the effect of their propagation in the time-dependent Higgs background.

We have shown that the Dirac equation for the electron field is equivalent to a second order differential equation with doubly-periodic coefficients. In the limit of very large primitive period of the Higgs background this equation can be solved in WKB approximation.

We summarize our findings by saying that the solutions to the Dirac equation in the slowly varying scalar-background field are plane-waves with a time-dependent distortion factor which becomes arbitrarily small in the limit of infinite primitive period of the Higgs vev. Therefore fermion fluctuations are almost like plane-waves with three-momentum \mathbf{p} and mass m_e but, strictly speaking, we have no time-dependent mass and \mathcal{E} of Eq.(41) is not the energy of the state; we rather register a departure from the plane-wave shape through a time-dependent factor which, of course, could be fitted with a plane-wave solution with an effective $m_e(t)$ parameter.

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